

Math 55, **First Midterm Exam**  
Thursday, February 26, 8:10am–9:30am

*This exam is closed book. You may not use any books, notes or electronic devices. Please write your answers in a blue note book. Write your name, the name of your TA and your section time on the cover. There are five problems, each worth 20 points, for a total of 100 points. Answers without justification will not receive credit. You may look at your graded exam in your discussion section on Monday, March 2.*

- (1) The value of the Euler  $\phi$ -function at a positive integer  $n$  is the number of positive integers less than or equal to  $n$  that are relatively prime to  $n$ . Compute the following four values of this function:  $\phi(36)$ ,  $\phi(37)$ ,  $\phi(81)$  and  $\phi(1024)$ .
- (2) Determine the truth value of each of these statement if the domain of each variable is the set of nonnegative integers:
  - (a)  $\exists x ((x^2 < 10) \wedge (|3 - x| > 2))$
  - (b)  $\forall x ((x \neq 4) \rightarrow (x - 5 > 1))$
  - (c)  $\forall x \exists y (x + y = 0)$
  - (d)  $\exists x \forall y (xy = 0)$
- (3) Prove that 5 divides  $n^5 - n$  whenever  $n$  is a positive integer.
- (4) Find an inverse of 81 modulo 250.
- (5) The symmetric difference  $A \oplus B$  of two sets  $A$  and  $B$  is the set containing those elements in either  $A$  or  $B$  but not in both  $A$  and  $B$ . Determine whether this operation associative; that is, if  $A$ ,  $B$  and  $C$  are sets, does it follow that  $A \oplus (B \oplus C) = (A \oplus B) \oplus C$  ?