

Math 55, **Second Midterm Exam**

Thursday, March 22, 8:10am–9:30am

This exam is closed book. You may not use any books, notes or electronic devices. Please write your answers in a blue note book. Begin by writing your name, the name of your GSI and your section time on the cover. There are five problems, each worth 20 points, for a total of 100 points. Answers without justification will not receive credit. You may look at your graded exam in your discussion section on Monday, April 2.

- (1) At a certain university, each student comes from precisely one of the 50 states. What is the minimum number of students who must be enrolled to guarantee that there are at least 100 who come from the same state?
- (2) Consider the three integers $65^{1000} - 8^{2001} + 3^{177}$, $19^{1212} - 9^{2399} + 2^{2001}$ and $24^{4493} - 5^{8192} + 7^{1777}$. Prove that the product of two of these numbers is nonnegative.
- (3) Suppose that Mike selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains 4 red balls and 3 blue balls, and the second box contains 2 red balls and 5 blue balls. What is the probability that Mike picked a ball from the second box if he has selected a red ball?
- (4) A fair die is rolled until the number 6 appears for the first time. After this happened, no further dice rolls are made.
 - (a) Find the probability that exactly four rolls are made.
 - (b) What is the expected number of dice rolls?
- (5) How many ways are there to distribute ...
 - (a) 8 indistinguishable balls into 5 distinguishable bins?
 - (b) 8 indistinguishable balls into 5 indistinguishable bins?