Chapter 1

Main topics

- Propositional logic
- Propositional equivalences
- Predicates and quantifiers
- Nested quantifiers
- Rules of inference (don't need to memorize names)

Types of problems

- Write truth tables for propositions
- Determine when two propositions are equivalent
- Determine when proposition is a tautology
- Turn English statements into propositions and vice-versa
- Determine truth values of propositional functions
- Be able to use rules of inference to build valid arguments

Chapter 2

Main topics

- Elements, sets, subsets; know difference between elements and sets
- Set operations (power set, Cartesian product, union, difference, complement)
- Functions (injective, surjective, bijective; inverses)
- Sequences (including arithmetic and geometric progressions)
- Summations

Types of problems

- Be able to prove set identities
- Be able to determine (with proof) whether a function is injective, surjective, bijective
- Be able to compute simple summations

Chapter 4

Main topics

- The integers (notion of divisor, relatively prime)
- Know definitions of $a \bmod m$ and $a \equiv b (\bmod m)$.
- Theorems about divisibility, including the Division algorithm
- Primes and composite numbers
- There are infinitely many primes
- Fundamental Theorem of Arithmetic
- Greatest common divison
- Modular arithmetic

Types of problems

- Be able to prove simple statements about divisibility (similar level of difficulty as on hw).
- Be able to convert numbers into base b expansion and back again.
- Be able to use the Euclidean algorithm to find the gcd.
- Given $a, b \in \mathbb{N}$, be able to write $gcd(a, b) = sa + tb$.
- Be able to find the inverse of $a \mod m$.
- Be able to solve systems of linear congruences using the Chinese Remainder Theorem.
- Be able to use Fermat's Little Theorem
- Understand how RSA works.

Chapter 5

Main topics

- Induction
- Strong induction
- Well-ordering principle
- Recursive definitions

Types of problems

- Proofs using induction and its variants, including proofs of statements about other topics we've covered in Math 55.

Chapter 6

Main topics

- Product and sum rules
- Permutations and combinations
- Combinatorial proofs
- Pigeonhole Principle (including Generalized Pigeonhole Principle)
- Binomial coefficients, Binomial Theorem, Pascal's identity

Types of problems

- Basic enumeration using product and sum rule
- Proofs using the pigeonhole principle
- Permutations with indistinguishable objects
- Distributing objects into boxes (star-bar problems)
- Proofs involving binomial coefficients, including combinatorial proofs.

Chapter 7

Main topics

- Probability (experiment, sample space, event, outcome, probability distribution)
- Conditional probability, independent events
- Bernoulli trials, the binomial distribution, geometric distribution
- Bayes' Theorem
- Sample space, random variable
- Independent random variables
- Expected value and variance of random variables (including linearity of expectation)

Types of problems

- Computing probabilities and conditional probabilities
- Applications of Bayes' Theorem
- Determining whether events and random variables are independent
- Computing expected values and variance of random variables
- Proofs of basic properties of expected value and variance.

Chapter 8

Main topics

- Recurrence relations, solving linear recurrence relations
- Generating functions
- Inclusion-exclusion and its applications

Types of problems

- Find the recurrence to describe a word problem.
- Solving linear homogeneous recurrence relations with constant coefficients by using the characteristic equation
- Find the closed form of the generating function of a sequence satisfying a recurrence
- Going from a generating function to an explicit formula for a sequence (using partial fractions
- Recognize basic generating functions (e.g. $\frac{1}{1-x}$, $\frac{1}{(1-x)^2}$)

- Be able to multiply generating functions (Theorem 1 from §8.4).
- Applications of inclusion-exclusion (e.g. derangements, counting solutions to an equation)

Chapter 9

Main topics

- Relations (including notions of reflexive, symmetric, antisymmetric, transitive)
- Combining relations (union, intersection, composition, etc.)
- Representing relations by matrices and directed graphs
- Closures of relations
- Equivalence relations; equivalence classes
- If R is an equivalence relation on a set S, its equivalence classes partition S

Types of problems

- Be able to recognize and prove when a relation has certain properties
- Understand representations of relations (matrices, directed graphs)
- Be able to compute closures of relations
- Be able to determine (with proof) whether a relation is an equivalence relation
- Be able to identify equivalence classes of relations; and determine how many there are

Chapter 10

Main topics

- Graphs and graph models
- Special types of graphs (complete, bipartite, wheel, etc)
- Representing graphs, graph invariants, graph isomorphism
- Paths and connectivity (for graphs, and for directed graphs)
- Eulerian circuits and paths

Types of problems

- Be able to determine sensible graph models
- Be able to determine when two graphs are isomorphic; whether a property is a graph invariant.
- Be able to determine whether a graph is connected; and whether a directed graph is strongly or weakly connected.
- Be able to determine the connected components of a graph
- Be able to determine whether a graph has an Eulerian circuit or path, and if so, find one.
- Proofs involving graphs