Final Exam Rubric and Comments

1. **Expected value:** 5 points if correct; 1 point if correctly defined, but not calculated.

   Variance: 5 points if correct; 4 w/ minor arithmetic error; 2 for answers of 9, 4.5, or 900 (see below); 1 if correctly defined, but not calculated or left as a massive summation.

   Total 7 points on this problem for an answer of $E(X) = 45$ and $V(X) = 4.5$ (disregarding that a "success" on Carmen's test is worth 2 points).

   Remarks. The most common mistake here was to assume that if Y if the number of correct answers Carmen gets, and $X = 2Y$ is her score, then $V(X) = 2V(Y)$. This leads to answer of $V(X) = 9$, but it is incorrect; the correct scaling rule is $V(aY) = a^2V(Y)$.

   The answer $V(X) = 900$ came from computing $V(X) = E(X^2) - E(X)^2$ under the mistaken belief that $E[(X_1 + \cdots + X_{50})^2] = E(X_1^2) + \cdots + E(X_{50}^2)$.

   The average score on problem 1 was 6.5 points.

2. 10 points for a correct result. 9 points for a minor error solving for coefficients, or for a correct result but a misunderstanding of "closed form". 8 points if indices offset by 1 ($a_n = 3^{n+1} + 3$). 4 points for the right characteristic equation but a malformed solution. 3 to 5 points for attempts to use generating functions that did not pan out.

   Remarks. It is hard to get through the generating function approach without making any mistakes; no one managed to do so.

   The average score on problem 2 was 8.7 points, making this the easiest problem.

3. **Injectivity:** 5 points for a proof, ≤ 2 if some claims unclear or dubious.

   Surjectivity: 5 points for a proof, 4 if referring to f^{-1} without justification, ≤ 2 for "g is injective so f is surjective."

   Total 4 points on this problem for "f and g are inverses so they must be bijections." While this is true, it is not much more than a restatement of the problem.

   The average score on problem 3 was 6.4 points.

4. **Part (a):** 5 points for a correct and justified answer. 3 if Alice and Bob probabilities both given correctly, but wrong choice of which is greater. 2 if only one of the two probabilities given correctly.

   Part (b): 5 points for a correct and justified answer. 2 if only Bob's expected value given correctly.

   The average score on problem 4 was 5.3 points.

5. 10 points for a correct result (FLT and CRT did not need to be cited by name if they were used straightforwardly). 4 points for using FLT correctly but not being able to finish. 2

points just for realizing FLT *could* be used, but no credit for solutions asserting that 55 is prime. No credit for attempted induction on a (the induction ladder has missing rungs...).

The average score on problem 5 was 6.5 points.

6. 5 points each part. On part (a), 2 points given for answers of $m+1$.

   The average score on problem 6 was 3.5 points, making this the hardest problem.

7. 10 points for a correct result. 9 points for direct approach w/ a minor arithmetic error. 6 to 8 for attempted complete expansion with one or more computation errors. 6 points for mistakes leading to the answers $\binom{15}{5}$, $\binom{14}{5}$, and $\binom{7}{4}$ (see below). 6 points for attempted PIE with mistakes.

   Remarks. The most common mistake was to find the coefficient of x^{10} in $(1+x+x^2+\cdots)^6$, which is $\binom{15}{5}$. In terms of the equivalent balls-and-boxes problem, this is ignoring the requirement that every box contain at least 1 ball. The answer of $\binom{14}{5}$ came from factoring out x instead of x^6 as a first step. The answer of $\binom{7}{4}$ came from confusing the number of balls with the number of boxes in the equivalent balls-and-boxes problem.

   The average score on problem 7 was 7.8 points.

8. **Part (a):** 1 point for reflexivity, 2 for symmetry, 3 for transitivity.

   Part (b): 4 points for the equivalence classes. 2 if only one class identified correctly. 1 if classes were identified in a complicated way (e.g. involving logs) or a way not specific to R (i.e. stating the definition of an equivalence class).

   Some solutions blurred the lines between parts (a) and (b), asserting that $m \; R \; n$ if and only if m, n have the same parity and that R is therefore obviously an equivalence relation. Such solutions generally got 6 points total.

   The average score on problem 8 was 5.7 points.

9. 10 points for a correct result. 8 if letters miscounted but approach otherwise sound. 7 for misunderstanding the problem as counting anagrams starting with H and ending with G (and doing so correctly). 4 for incorrect solutions that showed some appropriate use of casework or multinomial coefficients.

   The average score on problem 9 was 8.2 points.

10. 10 points for a correct proof. 9 if $>$ used where $\geq$ needed. 9 if degree zero vertices not ruled out. 8 if "$\sum \deg(v) \geq 2n \implies \geq n$ edges" stated but justified in an inappropriate way. 4 for attempted induction if it is not clear that all G can be reached via the inductive step. 2 for almost any attempt to engage with the problem (this was a common score).

    Remarks. Some people noticed that if G is connected and $1 \leq m \leq n-1$, then, in fact, $m = n-1$. This did not seem to make a difference in any solutions, however.

    The average score on problem 10 was 4.9 points.