Math 55: Discrete Mathematics, Fall 2008
Final Exam

1. Does there exist a 1-to-1 and onto function from the set of positive integers to the set of all integers? If so, construct one. If not, explain why not.

2. (a) Given the information that $2^{23376} \equiv 1 \pmod{23377}$, what, if anything, can you conclude about whether 23377 is prime?

(b) How would your answer change given the additional information that $2^{11688} \equiv 15907 \pmod{23377}$? [Note that $11688 = 23376/2$.]

3. Prove that in any set of ten integers there are two that differ by a multiple of 9.

4. A biased coin has probability $1/3$ of coming up heads on each flip. Let the random variable X be the number of heads seen in 99 independent flips of the coin.

(a) Find the expectation EX.

(b) Find the variance $V(X)$.

(c) Use the expectation and variance of X to find an upper bound on $P(X \geq 40)$.

5. Consider the linear code over $\mathbb{Z}_2$ given by the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(a) Find the minimum weight of a non-zero codeword in this code.

(b) How many errors can this code correct?

(c) Can this code detect more errors than it can correct?

6. Find the number of permutations of the 26 letters of the alphabet that do not contain any of the strings RUN, WALK, or SWIM in consecutive positions.

7. Consider the relation on the set of integers defined by $x \sim y$ if $x^2 \equiv y^2 \pmod 5$.

(a) Show that $\sim$ is an equivalence relation.

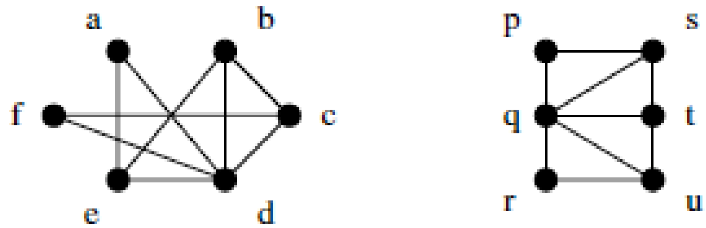
(b) Find its equivalence classes.

8. Let P be the set of integers $\{2, 3, 4, 6, 8, 12\}$, partially ordered by the divisibility relation $x \mid y$.

(a) Draw the Hasse diagram of P.

(b) Find the number of extensions of P to a compatible linear ordering.

9. Either find an isomorphism between the two graphs shown below, or show that none exists.



10. Prove that for all $n \geq 4$, the Stirling number $S(n, n-2)$ is given by the formula

$$S(n, n-2) = \binom{n}{3} + \frac{1}{2}\binom{n}{2, 2, n-4}.$$