

Math 55 Final Exam

Summer 2014

Name: _____

Problem	Score	Possible
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Σ		100

Note: Justify all answers.

Your answers on this test may include expressions of the form $n!$, n^k , and $\binom{n}{k}$. For example, if an answer is $52!$, you may leave it in that form; you don't need to write it out as a 68-digit number.

False name: _____

If you want your exam score posted to the course website under a false name, please write that name on the line above. If you leave this blank, your score will not be posted to the course website.

1. A true/false test has 50 questions, each worth 2 points. Carmen has a 90% chance of answering each question correctly (and her performance on each question is independent from her performance on other questions).

What are the expected value and variance of Carmen's total score?

2. If $a_1 = 6$, $a_2 = 12$, and $a_n = 4a_{n-1} - 3a_{n-2}$ for $n \geq 3$, solve for a_n in closed form.

3. Let $f : A \rightarrow B$ and $g : B \rightarrow A$ be functions such that $g(f(x)) = x$ for all $x \in A$ and $f(g(x)) = x$ for all $x \in B$. Show that f must be a bijection.

4. Each deck of cards in this problem is a well-shuffled standard 52-card deck with 13 hearts.

Alice draws 5 cards from 1 deck. Bob draws 1 card from each of 5 decks.

(a) Who has greater probability of drawing 5 hearts? Check one box:

☐ Alice ☐ Bob ☐ Equal

Show your work here:

(b) Who draws a greater expected number of hearts? Check one box:

☐ Alice ☐ Bob ☐ Equal

Show your work here:

5. Let a be an integer such that $\gcd(a, 55) = 1$. Show that $a^{20} \equiv 1 \pmod{55}$.

6. Let $m \leq n$ and let $K_{m,n}$ denote the complete bipartite graph on sets of m and n vertices.
- (a) How many distinct vertices must be chosen from $K_{m,n}$ to ensure that some two adjacent vertices are chosen?

7. Determine the coefficient of x^{10} in the expansion of $(x + x^2 + x^3 + \cdots)^6$.

8. Let R be the relation on $\mathbb{N}$ such that $m R n$ if and only if 3 divides $2^m - 2^n$.
- (a) Show that R is an equivalence relation.

(b) What are the equivalence classes?

9. How many anagrams of HEDGEHOG begin and end with the same letter?

(An anagram is an arrangement of all the letters. It does not need to be an English word.)

10. Let G be an undirected graph with n vertices and m edges, where $1 \leq m \leq n-1$. Suppose G is connected. Show that G has a vertex of degree 1.

(Scratch paper)

(Scratch paper)