

Math 55 Midterm Exam #1

Summer 2014

Name: _____

Problem	Score	Possible
1		20
2		20
3		20
4		20
5		20
Σ		100

1. Mark each statement below true or false. Give a short reason for each of your responses.

(a) $\forall x \in \mathbb{R} \exists y \in \mathbb{R} (x^2 - y^2 = 1)$

(b) $\forall y \in \mathbb{R} \exists x \in \mathbb{R} (x^2 - y^2 = 1)$

(c) $p \leftrightarrow q$ is logically equivalent to $\neg p \leftrightarrow \neg q$

(d) $[\exists x P(x)] \wedge [\exists x Q(x)]$ is logically equivalent to $\exists x [P(x) \wedge Q(x)]$

2. Let A, B be sets. Show that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

3. Let x, y, z are integers, no two of which are equal.

Show that if $x + y + z = 13$, then $\max(x, y, z) \geq 6$.

4. (a) Find an inverse of 7 modulo 100.

(b) Consider the function $f : \{0, 1, 2, \dots, 98, 99\} \rightarrow \{0, 1, 2, \dots, 98, 99\}$ defined by $f(n) = 7n \bmod 100$. For example, $f(20) = 40$.

Determine, with justification, whether f is injective and whether f is surjective.

5. Determine the smallest **two** solutions of the system

$$\begin{cases} n \equiv 1 \pmod{5} \\ n \equiv 3 \pmod{7} \\ n \equiv 8 \pmod{9} \end{cases},$$

where $n \in \mathbb{Z}^+$.